

Dynamics and Duality of a Stabilized Radion

Chris Verhaaren

Fermilab Theory Seminar

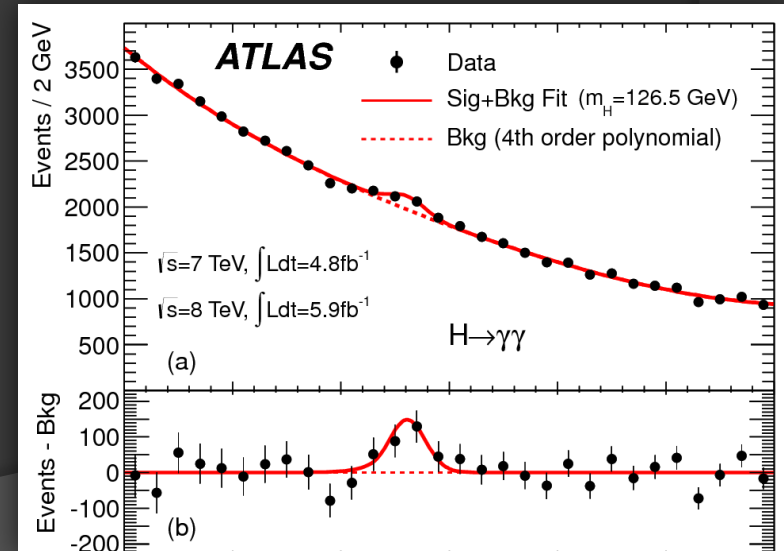
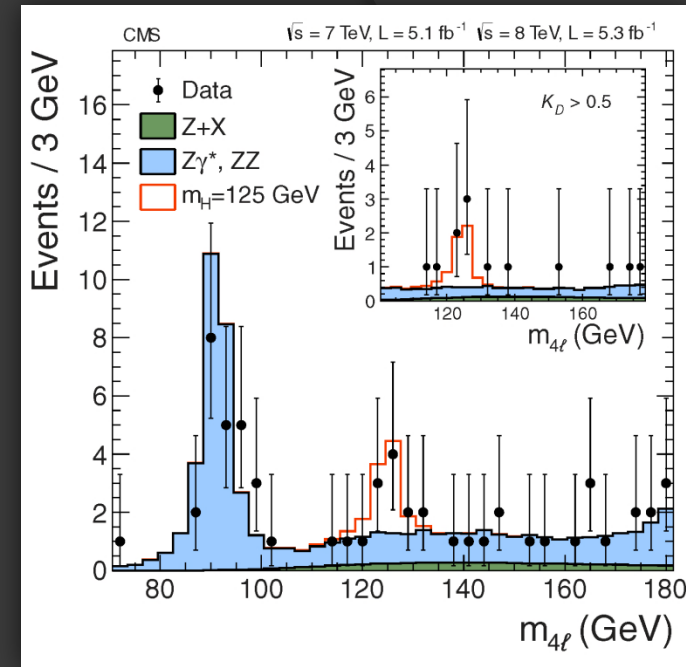
31 July 2014

arXiv: 1408.xxxx

Zackaria Chacko
Rashmish Mishra
Daniel Stolarski

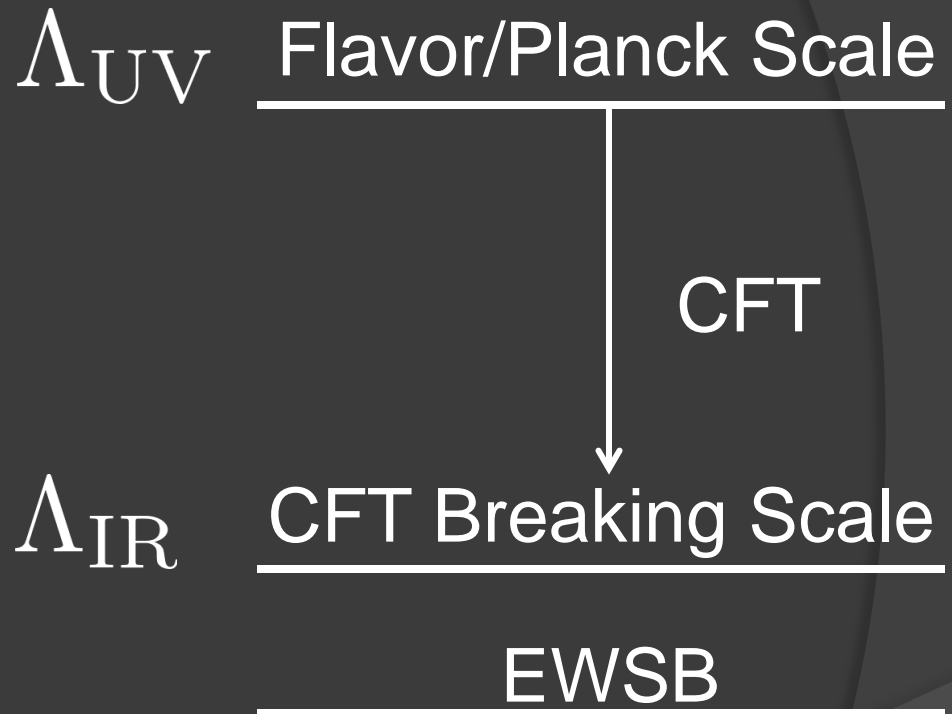
We have a Higgs!

- Next, understand EWSB
- Several possibilities address Planck-Weak hierarchy
- Two Roads: Elementary or Composite Higgs
- If composite, we must separate the weak scale from the flavor scale



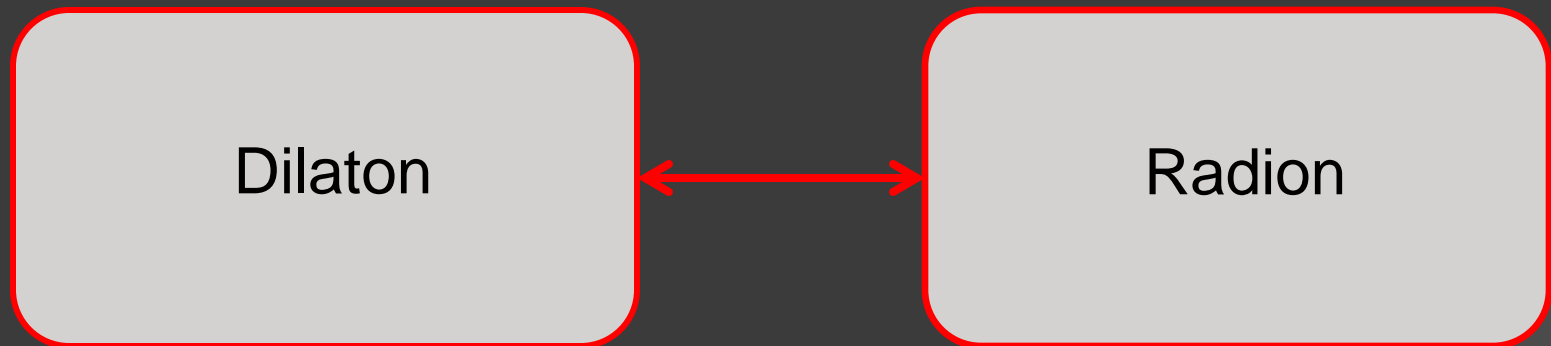
Conformal Models

- Conformal dynamics can be used in composite Higgs models to separate EW and flavor scales
- RS models are dual to strongly coupled conformal dynamics through AdS/CFT
- RS models contain the Radion



Outline

- Begin by recounting general known results about the dilaton of broken CFTs
- We then use these general results to understand the RS radion
- Great case study for AdS/CFT



The Dilaton σ

- The Standard Model is not conformal, so the symmetry must be broken, at scale f
- If the conformal symmetry was exact the low energy theory contains 1 massless NGB: the dilaton
 - Although 5 generators are broken only one NGB
- Couplings fixed to realize the broken symmetry nonlinearly
 - Tightly constrained

Explicitly Violated

- There is no such massless scalar field, so the conformal symmetry is explicitly broken
- The dilaton becomes massive
- Its couplings are corrected
- Two Questions
 - What is its mass?
 - How do its couplings change?

CFT \leftrightarrow RS

- Answers are known in the CFT case
- The radion of Randall-Sundrum models with SM fields in the bulk is dual to the dilaton of a CFT with partially composite SM fields
- We will calculate the couplings of a stabilized radion and show how they agree with the CFT analysis

(Massless) Dilaton Properties

- Under $x \rightarrow e^{-\omega} x$ the dilaton shifts

$$\sigma \rightarrow \sigma + f\omega$$

- Where f is the breaking scale

- It is convenient to define

$$\chi = f e^{\sigma/f}$$

- Which transforms as

$$\chi \rightarrow e^{\omega} \chi$$

Dilaton Couplings

- Consider gauge bosons that weakly gauge a global symmetry of the CFT (dual to gauge bosons in bulk of RS)
- Dominant coupling to σ comes from the non-scale invariant mass term

– Choose the basis in which $-\frac{1}{4g^2}F^2$

$$d^4x \frac{m_W^2}{g^2} W_\mu W^\mu \rightarrow e^{-2\omega} d^4x \frac{m_W^2}{g^2} W_\mu W^\mu$$

Dilaton Couplings

- Need two powers of $\frac{\chi}{f}$ to compensate

$$\left(\frac{\chi}{f}\right)^2 \frac{m_W^2}{g^2} W_\mu W^\mu \Rightarrow 2 \frac{\sigma}{f} \frac{m_W^2}{g^2} W_\mu W^\mu$$

- Looks Higgs-ish

Dilaton Potential

- The χ Lagrangian contains the usual derivative terms
- Because $d^4x \rightarrow e^{-4\omega} d^4x$ find unique potential

$$V(\chi) = \kappa_0 \chi^4$$

- Then κ_0 determines f
 - If $\kappa_0 > 0$ then $f = 0$. Symmetry never broken
 - If $\kappa_0 < 0$ then $f \rightarrow \infty$. Never conformal
 - If $\kappa_0 = 0$ then f is unconstrained. Tuned choice

Nearly Conformal

- Consider a deformed CFT with 1 relevant deformation

$$\mathcal{L}_{\text{CFT}} + \lambda_{\mathcal{O}} \mathcal{O}$$

- Grows in the IR until symmetry is broken
- Gives a mass to the dilaton, changes couplings
- To get the required big hierarchy between EW and flavor scales, we need $\Delta_{\mathcal{O}} = 4 - \epsilon$

2 Conformal Limits

- If $\lambda_{\mathcal{O}} \rightarrow 0$ then we have an exact CFT
- This is the standard story
- However, if $\Delta_{\mathcal{O}} \rightarrow 4$ we also recover an exact CFT
- With $\Delta_{\mathcal{O}} = 4 - \epsilon$ we still have a small parameter even if $\lambda_{\mathcal{O}}$ is large
- Can use both limits to understand dilaton mass and couplings

A Light Dilaton

- Most interested in dilatons we can discover
- The mass of the dilaton can be lighter than the CFT breaking scale if explicit breaking is small
 - The deformation is small at the breaking scale
$$\hat{\lambda}_{\mathcal{O}} \equiv f^{-\Delta_{\mathcal{O}}} \lambda_{\mathcal{O}} \ll 1$$
 - The deforming operator is nearly marginal at the breaking scale $\epsilon \ll 1$

Dilaton Mass

- Make $\lambda_{\mathcal{O}}$ a spurion to trace conformal symmetry violation

$$\mathcal{L}_{\text{CFT}} + \lambda_{\mathcal{O}} \mathcal{O} \qquad \lambda_{\mathcal{O}} \rightarrow e^{(4-\Delta_{\mathcal{O}})\omega} \lambda_{\mathcal{O}}$$

- To leading order in $\lambda_{\mathcal{O}}$

$$V(\chi) = \kappa_0 \chi^4 + \kappa_1 \lambda_{\mathcal{O}} \chi^{\Delta_{\mathcal{O}}}$$

- Leads to dilaton mass

Symmetry Breaking Parameters

$$m_{\sigma}^2 = 4f^2 \kappa_0 (4 - \Delta_{\mathcal{O}}) \sim (4\pi f)^2 \hat{\lambda}_{\mathcal{O}}(f) \epsilon$$


A More Correct Analysis

- Because the CFT is deformed, $\hat{\lambda}_{\mathcal{O}}$ runs
$$\frac{d \ln \hat{\lambda}_{\mathcal{O}}}{d \ln \mu} = -g(\hat{\lambda}_{\mathcal{O}}) = -\sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n$$
- With $c_0 = -\epsilon$ and all other $c_i \sim 1$
- Including this effect we find

$$m_{\sigma}^2 \sim (4\pi f)^2 \hat{\lambda}_{\mathcal{O}}(f) g(\hat{\lambda}_{\mathcal{O}}(f))$$

↑ ↑
Symmetry Breaking
Parameters

Light Dilaton Not Generic

- What changes from $\hat{\lambda}_O \epsilon \rightarrow \hat{\lambda}_O g(\hat{\lambda}_O)$?
- Typically the CFT breaks when $\hat{\lambda}_O \sim 1$
- In this case $g(\hat{\lambda}_O) \sim \hat{\lambda}_O \gg \epsilon$ so the mass is of the order of the breaking scale
- When is the dilaton light?
 - Naturally Light: Fixed Lines, Conformal Window
 - Tuning to break before $\hat{\lambda}_O \sim 1$
 - Tuning is linear, mild

Corrected Couplings

- Consider the gauge boson mass term

$$\left(\frac{\chi}{f}\right)^2 \left[1 + \alpha_W \hat{\lambda}_O \chi^{\Delta_O - 4}\right] \frac{\hat{m}_W^2}{\hat{g}^2} W_\mu W^\mu$$

- Leads to correction to the boson mass

$$\frac{m_W^2}{g^2} = \left[1 + \alpha_W \hat{\lambda}_O(f) f^{\Delta_O - 4}\right] \frac{\hat{m}_W^2}{\hat{g}^2}$$

Corrected Couplings

- This leads to

$$\frac{\sigma}{f} \left[2 + c_W \epsilon \hat{\lambda}_{\mathcal{O}}(f) \right] \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-}$$

Symmetry Breaking Parameters

Corrected Mass

- Exact form from dilaton mass!

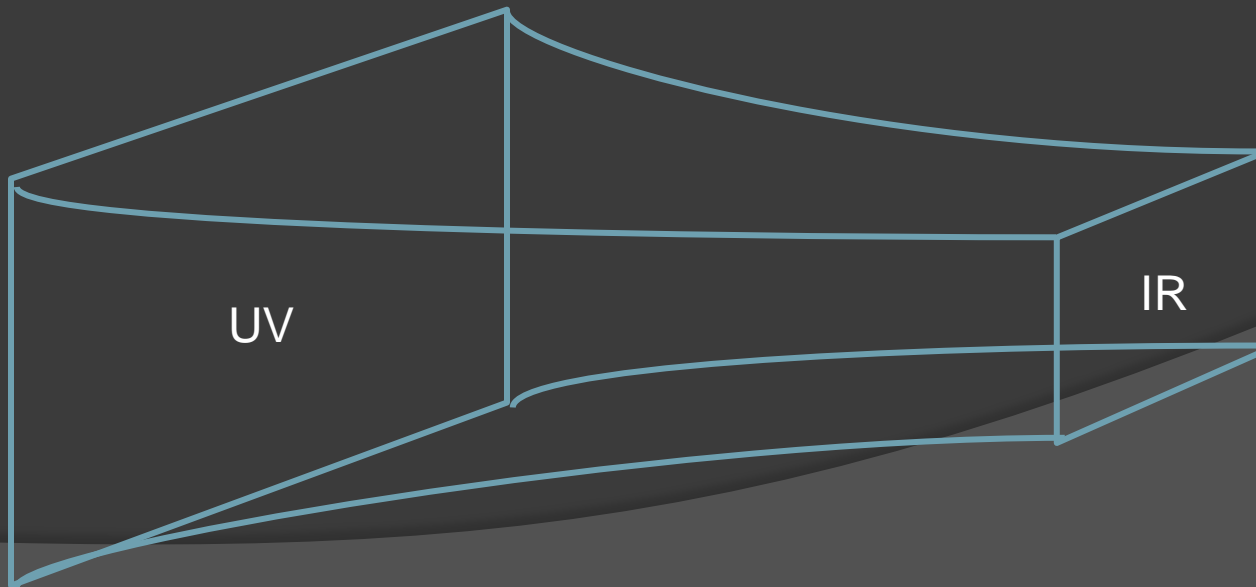
- Corrections goes like $\frac{m_\sigma^2}{\Lambda_{\text{IR}}^2}$

The Dilaton Story

- Realistic models predict a massive dilaton
- Theories with a light dilaton are somewhat special, but phenomenologically interesting
- Couplings are dictated by conformal symmetry with corrections of order $\frac{m_\sigma^2}{\Lambda_{\text{IR}}^2}$

Randall-Sundrum Models

- One additional spatial dimension which is highly curved
- With SM fields in the bulk, addresses the gauge hierarchy problem and fermion mass hierarchy puzzle



RS Models

- New states:
 - Kaluza-Klein modes
 - The Radion
- Related to a strongly coupled CFT in four dimensions by AdS/CFT

RS Geometry

- RS Gravity Action

$$S = \int d^4x d\theta \left[\sqrt{G} (-2M_5^3 \mathcal{R}_5 - \Lambda_b) \right. \\ \left. - \sqrt{-G_{UV}} \delta(\theta) T_{UV} - \sqrt{-G_{IR}} \delta(\theta - \pi) T_{IR} \right]$$

- With metric

$$ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2 \quad -\pi \leq \theta \leq \pi$$

- We parameterize the radion by letting

$$r_c \rightarrow r(x)$$

Radion Potential

- Integrate over the extra dimension to find

$$\int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right)$$

- With radion field

$$\varphi(x) = F e^{-k\pi r(x)} \quad F = \sqrt{\frac{24M_5^3}{k}}$$

- And potential (it's quartic! Think duality)

$$V_{\text{GR}}(\varphi) = \frac{\varphi^4}{F^4} \left(T_{\text{IR}} - \frac{\Lambda_b}{k} \right) \Leftrightarrow \kappa_0 \chi^4$$

Radion Phenomenology

- Couplings dictated by general covariance
 - Look Higgs-ish
- Tuning issue in the potential
 - If $kT_{\text{IR}} > \Lambda_b$ then $\langle r \rangle \equiv r_c \rightarrow \infty$
 - If $kT_{\text{IR}} < \Lambda_b$ then $r_c \rightarrow 0$
 - If $kT_{\text{IR}} = \Lambda_b$ then r_c is unconstrained. Tuned.

$$V_{\text{GR}}(\varphi) = \frac{\varphi^4}{F^4} \left(T_{\text{IR}} - \frac{\Lambda_b}{k} \right)$$

Goldberger-Wise Mechanism

- The radius r_c can be stabilized by a bulk scalar

$$S = \int d^4x d\theta \left[\sqrt{G} \left(\frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sqrt{-G_{UV}} \delta(\theta) V_{UV}(\Phi) - \sqrt{-G_{IR}} \delta(\theta - \pi) V_{IR}(\Phi) \right]$$

- In general Φ depends on $r(x)$ and contributes to the radion potential and leads to stabilization

Equation for Φ

- Φ satisfies

$$\partial_\theta^2 \Phi - 4kr_c \partial_\theta \Phi - r_c^2 \frac{dV_b}{d\Phi} = 0$$

- In general

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{3!}\eta\Phi^3 + \dots$$

- Including any but the mass term leads to a nonlinear ODE even neglecting back reaction

RS Holography

CFT

RS

UV Cutoff

\Leftrightarrow

UV Brane

Symmetry Breaking Scale

\Leftrightarrow

IR Brane

RG Scale, μ

\Leftrightarrow

Warped Dimension, θ

Dilaton, σ

\Leftrightarrow

Radion, φ

Explicit Symmetry Breaking, $\lambda_{\mathcal{O}}$ \Leftrightarrow

GW Scalar, Φ

Φ Holography

- Equation governing $\hat{\lambda}_{\mathcal{O}}$ is first order

$$\frac{d \ln \hat{\lambda}_{\mathcal{O}}}{d \ln \mu} = -g(\hat{\lambda}_{\mathcal{O}})$$

- Equation governing Φ is second order

$$\partial_{\theta}^2 \Phi - 4kr_c \partial_{\theta} \Phi - r_c^2 \frac{dV_b}{d\Phi} = 0$$

- How can $\lambda_{\mathcal{O}}(\mu)$ and $\Phi(\theta)$ be dual to each other?

Φ Holography

$$\cancel{\partial_\theta^2 \Phi} - 4kr_c \partial_\theta \Phi - r_c^2 \frac{dV_b}{d\Phi} = 0$$

- For a large hierarchy $kr_c \gg 1$ and so

$$\frac{d\Phi}{d\theta} = -\frac{r_c}{4k} \frac{dV_b}{d\Phi} \left(0 \leq \theta \leq \pi - \frac{1}{4kr_c} \right)$$

- Or

$$\frac{d \ln \Phi}{d(kr_c \theta)} = -\frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}} \frac{\Phi}{k^{3/2}} - \dots$$

Φ Holography

- Value of $\hat{\lambda}(\mu)$ dual to $k^{-3/2}\Phi(\theta)$
- So
$$\frac{d \ln \hat{\lambda}}{d \ln \mu} = \epsilon - c_1 \hat{\lambda} - \dots$$
- is dual to
$$\frac{d \ln \Phi}{d(kr_c \theta)} = -\frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}} \frac{\Phi}{k^{3/2}} - \dots$$
- Mass of Φ dual to scaling dimension of \mathcal{O}

$$\Delta_{\mathcal{O}} = 4 - \epsilon = 4 + \frac{m^2}{4k^2}$$

The Small Parameters

$$\hat{\lambda}_O \rightarrow k^{-3/2} \Phi \quad \hat{\lambda}_O g(\hat{\lambda}_O) \Leftrightarrow k^{-3/2} \frac{d\Phi}{d(kr_c \theta)}$$

$$g(\hat{\lambda}_O) \rightarrow \frac{d \ln \Phi}{d(kr_c \theta)}$$

- Consider the constant Φ limit

- Parameters rescaled
- No stabilization

- Radion mass depends on $\frac{d\Phi}{d(kr_c \theta)}$
- In theories of interest $\frac{d\Phi}{d(kr_c \theta)} \ll 1$ until near the breaking scale

Radion Mass

- Dual to dilaton, mass is generically of order the KK scale
- Natural light mass if Φ a pNGB (dual to a quasi-fixed line)
- Other theories with a light Radion employ mild tuning
- In all light mass cases, Φ is a slowly varying function:

$$\frac{d\Phi}{d(kr_c\theta)} \ll 1$$

Radion Couplings after Stabilization

- Will show corrections to couplings to bulk SM fields
- Demonstrate how holography organizes the results
 - Correction that do not affect the form of the dilaton couplings
 - Corrections to the form
- Corrections to the form are of order $\frac{m_\varphi^2}{m_{\text{KK}}^2}$

Massive Gauge Bosons

- Mass term

$$S = \int d^4x d\theta \delta(\theta - \pi) \sqrt{-G_{\text{IR}}} G_{\text{IR}}^{\mu\nu} (\mathcal{D}H)(\mathcal{D}H)^\dagger$$

$$\Rightarrow \int d^4x e^{-2kr(x)\pi} W_\mu W^\mu \langle H \rangle^2$$

- Let $r(x) \rightarrow r_c + \delta r(x)$ or $\varphi(x) \rightarrow f + \tilde{\varphi}(x)$
with $-k\pi\delta r = \frac{\tilde{\varphi}}{f}$

Massive Gauge Bosons

- Then

$$S = \int d^4x e^{-2kr_c\pi} \langle H \rangle^2 W_\mu W^\mu \left(1 + 2 \frac{\tilde{\varphi}}{f} \right)$$

- So the coupling is

$$2 \frac{m_W^2}{g_4^2} \frac{\tilde{\varphi}}{f} W_\mu W^\mu$$

- Similar to the Higgs

↑
Basis Choice

Stabilized Massive Gauge Boson

- Now,

$$\int d^4x e^{-2kr(x)\pi} \langle H \rangle^2 W_\mu W^\mu \left(1 + \frac{\beta_W}{k^{3/2}} \Phi(\pi) \right)$$

- Note that the mass is altered, same as CFT
- As $r \rightarrow r_c + \delta r$, we find

$$\Phi(\theta k r) \rightarrow \Phi_c(\theta k r_c) + \delta r \theta k \Phi'_c$$

where

$$\Phi'_c \equiv \frac{d\Phi_c}{d(kr_c\theta)}$$

Stabilized Massive Gauge Boson

- Finally,

$$\int d^4x \frac{m_W^2}{g_4^2} W_\mu W^\mu \left[1 + \frac{\tilde{\varphi}}{f} \left(2 - \frac{\beta_W \Phi'_c(\pi)}{k^{3/2} + \beta_W \Phi_c(\pi)} \right) \right]$$

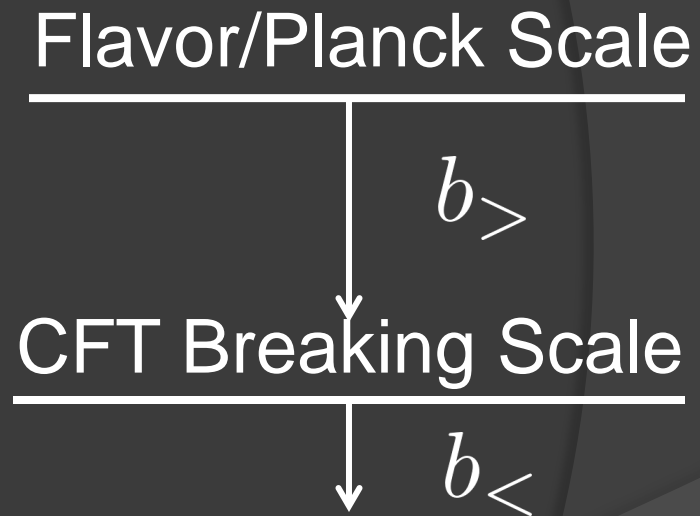
- Note the altered mass
- From duality

$$g(\hat{\lambda}_\mathcal{O}) \hat{\lambda}_\mathcal{O} \Leftrightarrow k^{-3/2} \Phi'_c$$

- Radion coupling correction goes like $\frac{m_\varphi^2}{m_{\text{KK}}^2}$

Gauge Boson Kinetic Term

- Bulk gauge bosons are dual 4D gauge fields that weakly gauge global symmetries of the CFT
- The kinetic term is classically conformal, but the gauge coupling runs above and below the breaking scale



$$\frac{d}{d \ln \mu} \frac{1}{g_{\text{UV,IR}}^2} = \frac{b_{>,<}}{8\pi^2} \quad \frac{b_{<} - b_{>}}{32\pi^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu}$$

Gauge Boson Kinetic Term

- Before stabilization

$$-\int d^4x d\theta \frac{F^2}{4} \left[\delta(\theta) \frac{\sqrt{-G_{UV}}}{g_{UV}^2} + \frac{\sqrt{G}}{g_5^2} + \delta(\theta - \pi) \frac{\sqrt{-G_{IR}}}{g_{IR}^2} \right]$$

- In KK decomposition the zero mode has a flat profile
- Integrate over θ to find

$$\frac{1}{g_4^2} = \frac{1}{g_{UV}^2} + \frac{2\pi r_c}{g_5^2} + \frac{1}{g_{IR}^2}$$

Gauge Boson Kinetic Term

- Integrate over θ and let $r \rightarrow r_c + \delta r$

$$-\int d^4x d\theta \frac{F^2}{4} \left[\delta(\theta) \frac{\sqrt{-G_{UV}}}{g_{UV}^2} + \frac{\sqrt{G}}{g_5^2} + \delta(\theta - \pi) \frac{\sqrt{-G_{IR}}}{g_{IR}^2} \right]$$

- Yields coupling $\frac{1}{2kg_5^2} \frac{\tilde{\varphi}}{f} F^2$

Loop Effects

- In the effective theory with $-\frac{1}{4g^2(\mu)}F^2$

$$g_4^2 = g^2(\Lambda_{\text{IR}} = ke^{-kr\pi})$$

- From the RGE

$$\frac{1}{g^2(\mu)} = \frac{1}{g_4^2} - \frac{b_{<}}{8\pi^2} \ln \frac{ke^{-kr\pi}}{\mu}$$

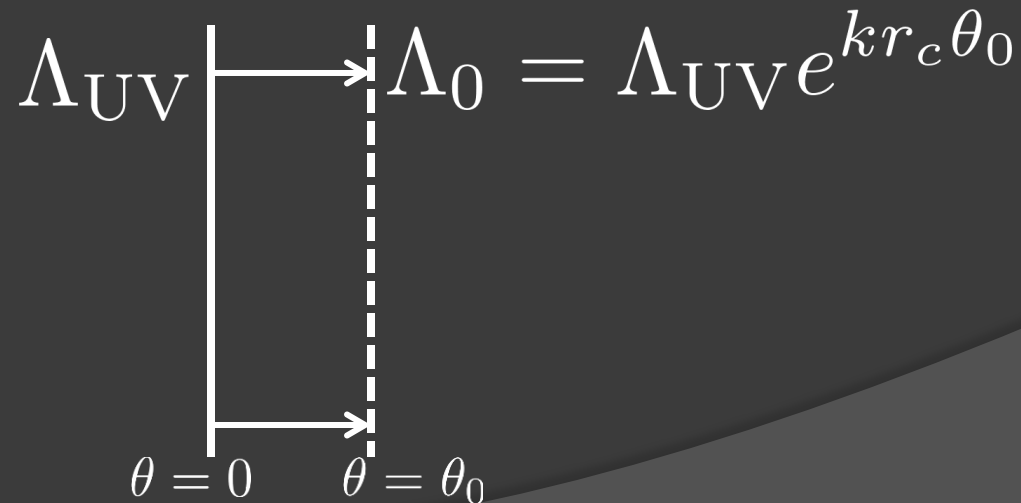
\swarrow
 $r + \delta r$

- Leads to dilaton coupling

$$\frac{b_{<}}{32\pi^2} \frac{\tilde{\varphi}}{f} F^2$$

Holographic Guide

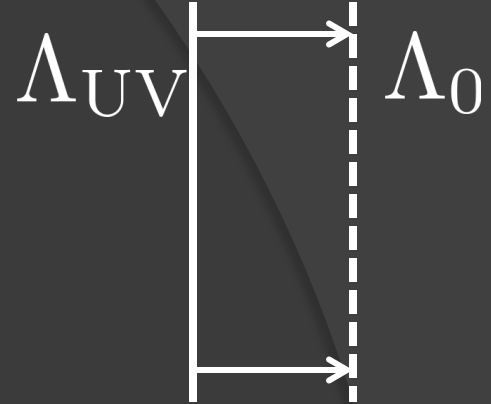
- To one loop $\left(\frac{1}{2kg_5^2} + \frac{b_{<}}{32\pi^2} \right) \frac{\tilde{\varphi}}{f} F^2$
- To relate to the CFT parameter $b_{>}$ move the UV brane



The diagram illustrates the relationship between the UV brane and the IR brane in a holographic setup. It features two vertical lines representing the branes. The left vertical line is solid and labeled Λ_{UV} at its top. The right vertical line is dashed and labeled Λ_0 at its top. Two horizontal arrows point from the left vertical line to the right vertical line: one at the top and one at the bottom. The bottom horizontal arrow is labeled $\theta = 0$ at its left end and $\theta = \theta_0$ at its right end. To the right of the dashed line, the equation $\Lambda_0 = \Lambda_{UV} e^{kr_c \theta_0}$ is written.

$$\Lambda_{UV} \longrightarrow \Lambda_0 = \Lambda_{UV} e^{kr_c \theta_0}$$

$\theta = 0 \qquad \theta = \theta_0$



Holographic Guide

- Integrating the action from 0 to θ_0
we find

$$\frac{1}{g^2(\theta_0)_{UV}} = \frac{1}{g_{UV}^2} + \frac{2\theta_0 r_c}{g_5^2}$$

- So

$$\frac{b_>}{8\pi^2} = \frac{d}{d \ln \Lambda_0} \frac{1}{g^2(\Lambda_0)} = -\frac{1}{kr_c} \frac{d}{d\theta_0} \frac{1}{g_{UV}^2(\theta_0)} = -\frac{2}{kg_5^2}$$

$$\left(\frac{1}{2kg_5^2} + \frac{b_<}{32\pi^2} \right) \frac{\tilde{\varphi}}{f} F^2 = \frac{b_< - b_>}{32\pi^2} \frac{\tilde{\varphi}}{f} F^2$$

Stabilized Coupling

- Add GW couplings

$$\frac{F^2}{4} \left[\delta(\theta) \frac{\sqrt{-G_{UV}}}{g_{UV}^2} \beta_{UV} + \frac{\sqrt{G}}{g_5^2} \beta_5 + \delta(\theta - \pi) \frac{\sqrt{-G_{IR}}}{g_{IR}^2} \beta_{IR} \right] \frac{\Phi}{k^{3/2}}$$

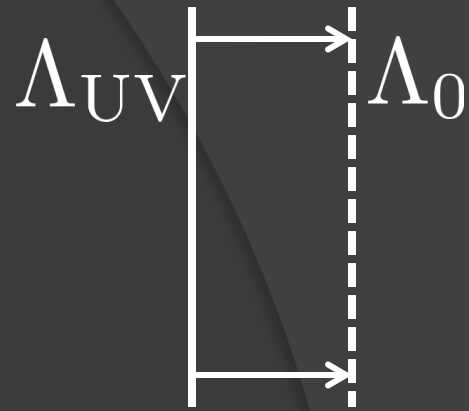
- To 1-loop, recalling $\Phi \rightarrow \Phi_c + \delta r \theta k \Phi'_c$

$$\left[\frac{1}{2k g_5^2} \left(1 + \frac{\beta_5}{k^{3/2}} \Phi_c(\pi) \right) + \frac{\beta_{IR}}{4g_{IR}^2 k^{3/2}} \Phi'_c(\pi) + \frac{b_{<}}{32\pi^2} \right] \frac{\varphi}{f} F^2$$

- Big correction!
What's going on?

$$\frac{m_\varphi^2}{m_{KK}^2}$$

Holographic Analysis



- Could this term be part of $b_>$?
- Move the UV brane to θ_0

$$\frac{1}{g_{UV}^2(\Lambda_0)} = \frac{1}{g_{UV}^2} + \frac{2r_c}{g_5^2} \int_0^{\theta_0} d\theta \left(1 + \frac{\beta_5}{k^{3/2}} \Phi_c(\theta) \right)$$

- Use RGE

$$\frac{b_>}{8\pi^2} = \frac{d}{d \ln \Lambda_0} \frac{1}{g^2(\Lambda_0)} = -\frac{1}{kr_c} \frac{d}{d\theta_0} \frac{1}{g_{UV}^2(\theta_0)}$$

Holographic Analysis

- Pushing the scale all the way to $\theta_0 = \pi$

leads to
$$\frac{b_{>}}{8\pi^2} = -\frac{2}{kg_5^2} \left(1 + \frac{\beta_5}{k^{3/2}} \Phi_c(\pi) \right)$$

$$\left[\frac{1}{2kg_5^2} \left(1 + \frac{\beta_5}{k^{3/2}} \Phi_c(\pi) \right) + \frac{\beta_{\text{IR}}}{4g_{\text{IR}}^2 k^{3/2}} \Phi'_c(\pi) + \frac{b_{<}}{32\pi^2} \right] \frac{\varphi}{f} F^2$$

- Becomes
$$\left[\frac{b_{<} - b_{>}}{32\pi^2} + \frac{\beta_{\text{UV}}}{4g_{\text{UV}}^2} \frac{\Phi'_c(\pi)}{k^{3/2}} \right] \frac{\varphi}{f} F^2$$

- Only corrections to the form are
$$\frac{m_\varphi^2}{m_{\text{KK}}^2}$$

Bulk Fermions

- Analysis proceeds as in gauge boson case
- As the profiles are not flat the analysis is more complicated
- Corrections to the scaling dimension of CFT operators by stabilization
- Corrections to the dilaton form by $\frac{m_\varphi^2}{m_{\text{KK}}^2}$

Conclusions

- Composite Higgs theories with CFT completion may possess a light dilaton
- Such theories are a result of *near* conformality
- This near conformality is small parameter
- Changes to radion couplings can be calculated and agree with dilaton analysis
- Corrections to massless dilaton form go like $\frac{m_\varphi^2}{m_{\text{KK}}^2}$